

# A dual order parameter from fundamentally colour charged matter\*

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The center phase transition of QCD and of fundamentally charged scalar QCD at non-vanishing temperature is investigated within a Dyson-Schwinger approach. The temperature dependence of the scalar/quark propagator is studied with generalized boundary conditions. A novel order parameter for the center phase transition is established which still exhibits a considerable dependence on the scalar/quark-gluon vertex.

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## 1. Motivation

During the last years, intensive theoretical and experimental efforts towards a deeper understanding of the confinement and chiral symmetry breaking phenomena have been made. In these proceedings we focus on the QCD phase transition at non-vanishing temperature and the related center symmetry breaking<sup>1</sup> by investigating so-called dual observables [3, 4, 5, 6, 7, 8]. Dual observables can be related to the spectrum of the Dirac operator and therefore comprehend information on both, the chiral properties and the confinement of quarks. They have first been introduced in lattice calculations [3] but are also accessible via functional methods [5, 6, 7, 8]. Within a Dyson-Schwinger approach [9] we study the center phase transition in QCD and in fundamentally charged scalar QCD. For the solution of the Dyson-Schwinger equation (DSE) of the scalar/quark propagator quenched lattice input for the gluon propagator has been used [7]. The main outcome of this investigation is a novel order parameter for the center phase transition [10] which still exhibits a considerable dependence on the used model for the

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<sup>1</sup> Center symmetry is realized only in the limit of infinitely heavy quarks while in real QCD the symmetry is always explicitly broken, cf. [1, 2] and references therein.

scalar/quark-gluon vertex. This requires a more detailed vertex analysis in future studies [11].

## 2. Center Phase Transition and Related Order Parameters

The DSE for the quark propagator at finite temperature reads<sup>2</sup>

$$S^{-1}(p) = Z_2 S_0^{-1}(p) - Z_2 C_F g^2 T \sum_{\omega_k(\phi)} \int \frac{d^3 k}{(2\pi)^3} \gamma^\mu S(k) \Gamma^\nu(k, p; q) D^{\mu\nu}(q) \quad (1)$$

with the momenta  $p = (\vec{p}, \omega_p)$ . The generalized Matsubara modes  $\omega_p(\phi) = (2\pi T)(n_p + \phi/2\pi)$  depend on the general boundary angle  $\phi \in [0, 2\pi)$ .<sup>3</sup> The usual fermionic boundary conditions are obtained for  $\phi = \pi$ . For the propagator we use the conventional ansatz  $S^{-1}(p) = i\gamma_4 \omega_p C(p) + i\vec{p}A(p) + B(p)$  and for the quark-gluon vertex the temperature dependent model [6]

$$\Gamma^\nu(k, p; q) = \tilde{Z}_3 \left( \delta^{4\nu} \gamma^4 \frac{C(k) + C(p)}{2} + \delta^{j\nu} \gamma^j \frac{A(k) + A(p)}{2} \right) \times \left\{ \frac{d_1}{d_2 + q^2} + \frac{q^2}{q^2 + \Lambda^2} \left( \frac{\beta_0 \alpha(\mu) \ln [q^2/\Lambda^2 + 1]}{4\pi} \right)^{2\delta} \right\}, \quad (2)$$

where  $k$  and  $p$  denote the in- and outgoing quark momenta and  $q$  the gluon momentum. The first part of the vertex (2) consists of a Slavnov-Taylor motivated ansatz, whereas the logarithmic tail together with the anomalous dimension  $2\delta = -18/44$  ensures a running coupling-like behaviour of the vertex in the UV regime. The remaining model parameters  $d_1$  and  $d_2$  are purely phenomenological, where in the following we take the values proposed in [7]. The system can now be solved by using the gluon propagator as input from quenched lattice results [7].

From the quark propagator (1), various order parameters can be extracted. The  $\phi$ -dependent quark condensate is given by

$$\langle \bar{\psi}\psi \rangle_\phi = Z_2 N_c T \sum_{\omega_p(\phi)} \int \frac{d^3 p}{(2\pi)^3} \text{tr}_D S(\vec{p}, \omega_p(\phi)) \quad (3)$$

<sup>2</sup> Here, for the quark-gluon vertex the renormalization factor  $Z_{1F} = Z_2/\tilde{Z}_3$  has been used, while in Landau gauge the ghost-gluon vertex renormalization constant  $\tilde{Z}_1 = 1$  [12]. Furthermore, the ghost renormalization constant  $\tilde{Z}_3$  is omitted as it cancels by a corresponding factor in the quark-gluon vertex model. The remaining constant  $Z_2$  is eliminated within a MOM scheme.  $C_F$  is the fundamental Casimir invariant,  $C_F = 4/3$  for  $N_c = 3$ .

<sup>3</sup> A similar approach has been employed in [8], where QCD with  $\phi$ -dependent boundary conditions for the quark fields has been referred to as  $QCD_\phi$ . This corresponds to QCD at imaginary chemical potential.

where the case  $\phi = \pi$  corresponds to the ordinary quark condensate. With a Fourier transform of the previous condensate (3) the dual quark condensate [3] is obtained with respect to the winding number  $n$

$$\Sigma_n = \int_0^{2\pi} \frac{d\phi}{2\pi} e^{-in\phi} \langle \bar{\psi}\psi \rangle_\phi. \quad (4)$$

In the following we fix  $n = 1$ .  $\Sigma_1$  transforms like the conventional Polyakov loop [1] under center transformation and is therefore a suitable order parameter for the center transition [3, 4, 5, 6, 7], cf. also [8]. Alternatively, the scalar quark dressing function  $B$ , evaluated at vanishing external momenta, is also sensitive to center symmetry breaking [6].<sup>4</sup> However, if only the lowest Matsubara mode is taken into account as suggested in [6] no satisfactory results could be obtained. Hence, we propose the following order parameter

$$\Sigma_Q = \int_0^{2\pi} \frac{d\phi}{2\pi} e^{-i\phi} \Sigma_{Q,\phi}, \quad \Sigma_{Q,\phi} = T \sum_{\omega_p(\phi)} \left[ \frac{1}{4} \text{tr}_D S(\vec{0}, \omega_p(\phi)) \right]^2 \quad (5)$$

for the center phase transition. The novel condensate  $\Sigma_{Q,\phi}$  is periodic in the boundary conditions and transforms like the conventional Polyakov loop under center transformations [10]. In contrast to the quark condensate  $\langle \bar{\psi}\psi \rangle_\phi$  the new condensate is also finite away from the chiral limit.

Before we substantiate our proposals with numerical results we proceed with the investigation of the center phase transition in scalar QCD.

### Fundamentally Charged Scalar QCD

Replacing the quarks by fundamental charged scalar fields provides a simpler model system due to the absence of Dirac structure. Neglecting the four-particle interactions the DSE for the scalar propagator reads<sup>5</sup>

$$D_S^{-1}(p) = \hat{Z}_3(p^2 + m^2) - \hat{Z}_3 C_F g^2 T \sum_{\omega_k} \int \frac{d^3k}{(2\pi)^3} (p+k)^\mu D_S(k) \Gamma_S^\nu(k, p; q) D_\gamma^{\mu\nu}(q) \quad (6)$$

with  $D_S(\vec{p}, \omega_p) = Z_S(\vec{p}, \omega_p)/(\vec{p}^2 + \omega_p^2)$ . For the scalar-gluon vertex we employ the model

$$\Gamma_S^\nu(k, p; q) = \tilde{Z}_3 \frac{D_S^{-1}(p^2) - D_S^{-1}(k^2)}{p^2 - k^2} (p+k)^\nu \times d_1 \left\{ \frac{\Lambda^2}{\Lambda^2 + q^2} + \frac{q^2}{q^2 + \Lambda^2} \left( \frac{\beta_0 \alpha(\mu) \ln[q^2/\Lambda^2 + 1]}{4\pi} \right)^{2\delta} \right\}. \quad (7)$$

<sup>4</sup> A related order parameter is the dual quark mass parameter  $\tilde{M}$  as proposed in [8].

<sup>5</sup> For details on the derivation and the truncation scheme see e.g. [13] and references therein. We have omitted renormalization constants which cancel due to our scalar-gluon vertex model. Additionally, we apply a MOM scheme to eliminate  $\tilde{Z}_3$ .

Here, the conventions for the momenta as well as the UV vertex behaviour is equivalent to the one in Eq.(2). In addition, we introduced a scalar propagator dependent function which is motivated from Ward identities in scalar electrodynamics and represents a generalized Ball-Chiu vertex [14]. The remaining part is again purely phenomenological where a dimensionless modeling constant  $d_1 = 0.53$  is introduced which improves the description of the phase transition. With the same lattice gluon propagator as in (1), the system can be solved<sup>6</sup> and the order parameters can be extracted.

Similar to the previous QCD case the following object serves as an order parameter for the center phase transition in scalar QCD

$$\Sigma_S = \int_0^{2\pi} \frac{d\phi}{2\pi} e^{-i\phi} \Sigma_{S,\phi}, \quad \Sigma_{S,\phi} = T \sum_{\omega_p(\phi)} D_S^2(\vec{0}, \omega_p(\phi)), \quad (8)$$

i.e.,  $\Sigma_{S,\phi}$  is periodic in  $\phi$  and transforms like the conventional Polyakov loop under center transformations [10].

### 3. Results

In order to confirm that  $\Sigma_Q$ , Eq.(5), and  $\Sigma_S$ , Eq.(8), are well-defined order parameters we consider both QCD and scalar QCD at finite temperatures. In Fig.1(a) the  $\phi$ -dependence of  $\Sigma_{Q,\phi}$  and the quark condensate  $\langle \bar{\psi}\psi \rangle_\phi$  are shown. While both quantities are periodic in  $[0, 2\pi)$  and symmetric in  $\phi \rightarrow -\phi$  the scalar quark dressing function  $B(p)$ , evaluated at vanishing momenta and lowest Matsubara mode, is not periodic. Above the critical temperature  $T_c = 277$  MeV all three quantities melt and, in the chiral limit, a plateau is formed. Below  $T_c$  the quark condensate is nearly constant while  $\Sigma_{Q,\phi}$  is slightly enhanced around  $\phi = \pi$ . This yields a non-zero value of  $\Sigma_Q$  below  $T_c$  as compared to the dual condensate  $\Sigma_1$  which almost vanishes in this region as shown in Fig.1(b). Note that the temperature behaviour of  $\Sigma_Q$  below the phase transition can be further tuned by varying the vertex model parameters. In contrast, the order parameter  $\Sigma_B$ , as defined in [6], considerably deviates from zero already below the phase transition, whereas  $\Sigma_1$  and  $\Sigma_Q$  stay close to zero. In order to see the expected behaviour of the order parameters the incorporation of all Matsubara modes is mandatory.

For scalar QCD we obtain similar results as is demonstrated in Fig.2(a) where the  $\phi$ -dependence of the condensate  $\Sigma_{S,\phi}$  is shown. Below  $T_c$  the condensate is  $\phi$ -independent and therefore  $\Sigma_S$  vanishes while above  $T_c$  it is suppressed around  $\phi = \pi$  and hence  $\Sigma_S$  is non-vanishing. This is displayed in Fig.2(b) where a distinct phase transition is observed. However, the results strongly dependent on the used scalar-gluon vertex model.

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<sup>6</sup> For this the program CrasyDSE [15] has been used.

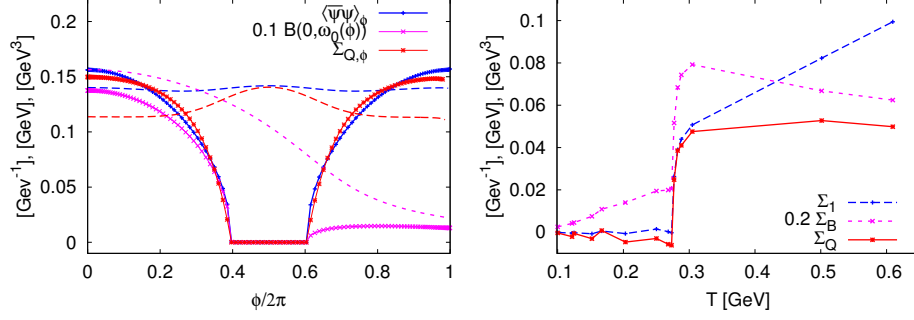


Fig. 1. Left panel: The quark dressing function  $B(\vec{0}, \omega_0(\phi))$ , the quark condensates  $\langle \bar{\psi}\psi \rangle_\phi$  and  $\Sigma_{Q,\phi}$ , as defined in Eq.(5), as a function of the boundary angle for different temperatures in the chiral limit (dashed lines:  $T = 273$  MeV, solid lines:  $T = 283$  MeV). Right panel: The order parameters  $\Sigma_Q$ ,  $\Sigma_1$  and  $\Sigma_B$  as defined in [6] as a function of the temperature.

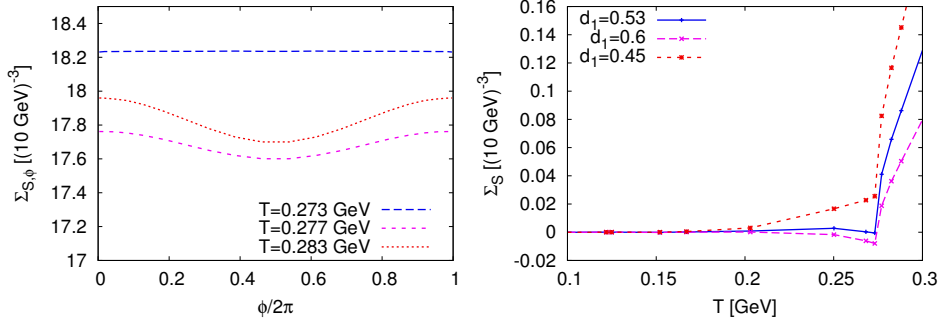


Fig. 2. Left panel:  $\Sigma_{S,\phi}$  as a function of the boundary angle for three different temperatures and  $m = 1.5$  GeV. Right panel: The order parameter  $\Sigma_S$  as a function of the temperature for three different vertex parameter values.

#### 4. Conclusions

Within the Dyson-Schwinger formalism we investigated the center phase transition of QCD and fundamentally charged scalar QCD. A novel order parameter was introduced which exhibits an improved behaviour below  $T_c$  compared to the order parameter  $\Sigma_B$  proposed in [6]. Motivated by the strong model dependence on the used vertex further studies of the corresponding scalar/quark-gluon vertex are on-going, see e.g. [11].

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